Lunar Rover

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will
- create a scale drawing to model a real life problem,
- apply the Pythagorean Theorem and distance/rate formula ($d = rt$), and
- analyze data to find a solution.

Prerequisites

Prior to this activity, students should have experience applying formulas. Students should be familiar with using calculators and evaluating formulas, as well as have a basic knowledge of scale factor and application of the Pythagorean Theorem.

Background

This problem applies mathematical principles in NASA's human spaceflight.

Exploration expands human presence into the solar system providing the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown; we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

In 1971, the Apollo 15 mission was the first mission to carry a lunar roving vehicle (LRV). This LRV (Figure 1) allowed astronauts to travel farther from their landing sites than in previous missions and explore and sample a much wider variety of lunar materials.

Because the vehicle was unpressurized, its longest single trip was 12.5 kilometers (7.8 miles). Its maximum range from the Lunar Module (LM) was 5.0 kilometers (3.1 miles). LRVs were also used during Apollo 16 and 17 missions in 1972.

If the LRV were to have failed at any time during the extravehicular activity (EVA), the astronauts had to have sufficient life support

Key Concepts
Pythagorean Theorem, distance/rate formula ($d = rt$)

Problem Duration
65 minutes

Teacher Prep Time
5–10 minutes

Technology
Computer with Internet access and projector, TI-84 Plus™

Materials
- Student Edition: *AL_ST_Lunar_Rover.pdf*
- Ruler or straight edge
- Graph paper
- Colored pencils
- Videos: *How Do Rovers on Mars?* and *Opportunity: Making Tracks on Mars*

Skill
Optimization, solving problems in a geometric context; creating data tables; applying formulas

NCTM Principles
- Algebra
- Geometry
- Problem Solving
- Communication
- Representation

Common Core Standards
- Geometry
- Algebra

consumables to be able to walk back to the LM. This distance is called the “walkback limit”, and it was approximately 10 kilometers (about 6 miles). Because of the reliability of the LRV and of the spacesuits, this restriction was relaxed on Apollo 17 for the longest traverse from the landing site—about 20 kilometers (or 12 miles).

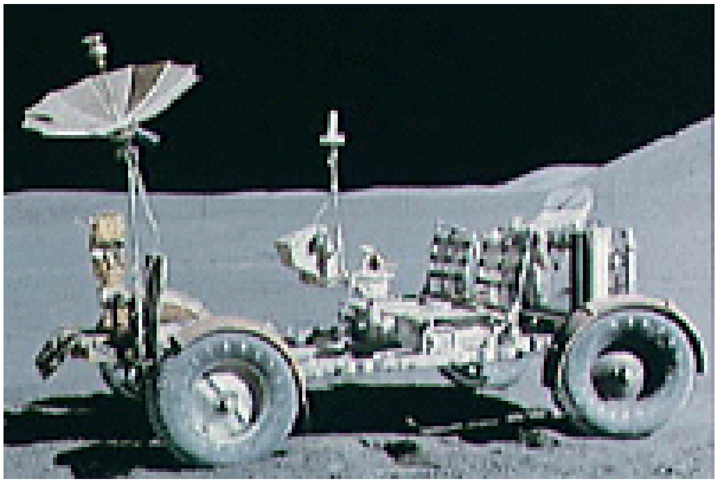


Figure 1: Apollo 15 Lunar Roving Vehicle taken on the Moon (NASA)

As NASA returns to the Moon and begins to explore other surfaces (such as Mars), exploration rovers will once again be needed to allow astronauts to traverse the terrain. Modern unpressurized rovers, which steer like cars, will look similar to those of the Apollo years (Figure 2) These vehicles will be limited to local travel of 10 to 20 kilometers (about 6 to12 miles) from the outpost site, where the astronauts would live on the Moon, for short periods of time (less than 10 hours), and will still require astronauts to wear space suits while traveling.

Figure 2: Unpressurized rovers (NASA concept)

A second roving vehicle concept, pressurized rovers, will give astronauts the ability to travel long distances (up to 200 kilometers) and perform extended science missions away from their habitat (Figure 3). They will provide a comfortable indoor environment for driving, while allowing the crew to use a variety of sensing and manipulation tools. These features will enable exploration and science to be performed without the need to exit the vehicle. Additional concepts for the pressurized rover include docking ports which allow the crew to directly enter the rover from their habitat and an airlock which permits extravehicular activity. The versatility of this rover, its power system, and its life support system will allow the astronauts to spend multiple Earth days away from their habitat performing work.

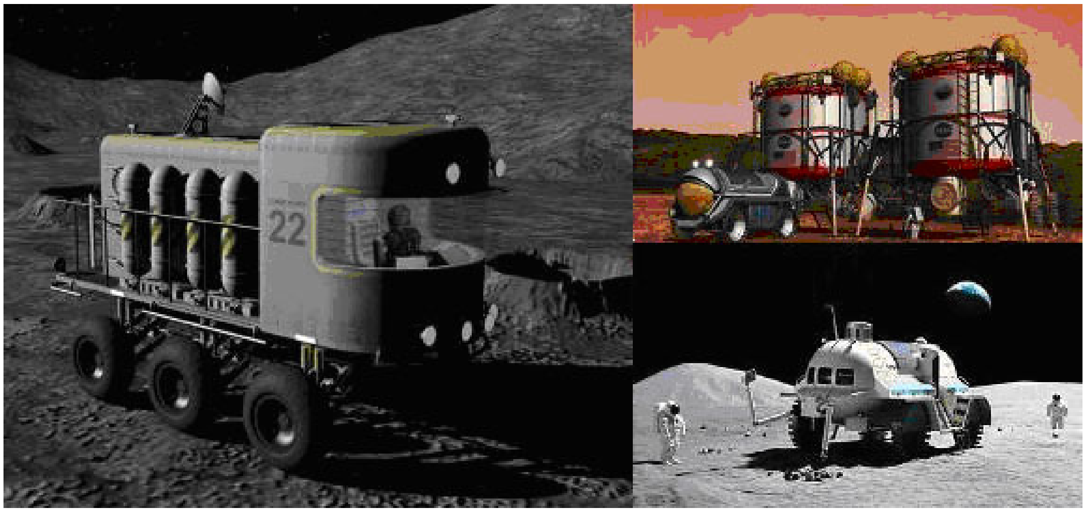


Figure 3: Pressurized rovers (Artist concepts)

NCTM Principles and Standards

Algebra

- Generalize patterns using explicitly defined and recursively defined functions
- Use symbolic Algebra to represent and explain mathematical relationships
- Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology
- Draw reasonable conclusions about a situation being modeled

Geometry

- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
- Use geometric models to gain insights into, and answer questions in, other areas of mathematics

Problem Solving

- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Representation

- Create and use representations to organize, record, and communicate mathematical ideas

Common Core Standards

Geometry

- Modeling with Geometry

Algebra

- Reasoning with Equations and Inequalities

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (15 minutes)

- Play the video, *How Do Rovers Drive on Mars?* (1:00 minute), accessible at the following link: http://www.nasa.gov/multimedia/videogallery/index.html?media_id=157212381
- Play the video, *Opportunity: Making Tracks on Mars* (3:18 minutes), accessible at the following link: http://www.nasa.gov/multimedia/videogallery/index.html?media_id=11463077
- With students in groups of three to four, ask them to review and discuss the main points of the background section for several minutes to be sure that they understand the material. Circulate to help facilitate discussion in small groups. Ask if any group needs clarification.

2 – Explore (10 minutes)

- Distribute copies of *Lunar Rover* Student Edition.
- Have students work in groups to answer questions 1.a–1.c.

3 – Explain (15 minutes)

- Have students work in groups to answer questions 1.d–1.i.
- Call on students to give their answers and discuss.

4 – Extend (15 minutes)

- Have students work in groups to answer questions 2.a–2.d.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (10 minutes)

- Have students work independently to answer question 3.
- This may be done in class or assigned as homework.

Lunar Rover

Solution Key

Have students read the background section. Then view the video. Encourage students to have a class discussion about the background and the video by asking questions about what has been seen and heard.

Some examples of questions for the educator to ask students are:

1. What are some of the main differences between the LRV and the Mars roving vehicles that are discussed in the videos?

   The LRV is manned, while Curiosity is not. LRV is not capable of traveling the same distances as the Mars roving vehicles. The Mars rovers are technologically advanced compared to the LRV.

2. Because the Mars Rover is not manned during its missions, how is the rover controlled?

   The rover follows commands that are entered into its computer by engineers. The rover uses cameras to guide itself safely to the destinations that are sent to the rover by the engineers.

3. What is the purpose of using vehicles like the LRV and Mars roving vehicles like Curiosity?

   The vehicles allow us to explore places beyond Earth like the Moon and Mars. The vehicles can be used to discover rocks and landforms on different planetary surfaces that can lead to possible discoveries of former life forms.

Directions: Read through the problem set-up and answer the following questions.

Problem

You are on the mission planning team that will determine the best crew route to be used on the first trip to the Moon using the new pressurized LRV (Rover 1). In this scenario, the crew must use Rover 1 to gather rock samples from around the deGerlache Crater.

On the map, Rover 1 is located at Habitat A (near Shackleton Crater). Crewmembers must drive Rover 1 to the rim of deGerlache Crater to collect rock samples. Rocks can be collected at any point along the edge of the crater (along segment $\overline{PQ}$ on the map). Before Rover 1 and the crew return to the habitat, they must also stop at point B on the map in order to reset a seismic sensor that has been gathering data about the interior of the Moon. All distances are denoted on the problem diagram (Figure 4).

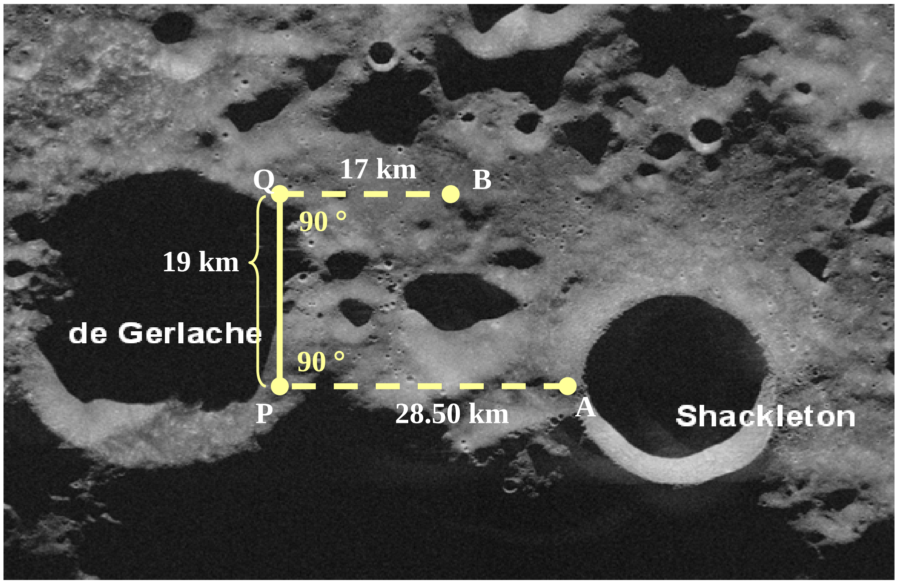


Figure 4: Problem diagram

Directions: Answer questions 1.a–1.c. in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

1. Minimal Path Problem

   Your mission planning team must find the shortest total path for the pressurized Rover 1 to travel for this exploration.

   - The crew will start at Habitat A and travel to a point along deGerlache Crater (on segment $\overline{PQ}$) to collect rock samples.
   - The crew will then travel to point B to reset the seismic sensor.

   a. Create a scale drawing of the mission on the graph provided (Graph 1).
      i. Point P is located at the origin
      ii. Segment $\overline{AP}$ lies on the x-axis
      iii. Segment $\overline{PQ}$ lies on the y-axis
      iv. Point B is located on the horizontal line through point Q
      v. Chose a point on $\overline{PQ}$ along the deGerlache Crater that you think would give the shortest total distance. Label that point G for "guess".

   b. The crew may choose to travel from point A to point P to point B. Use a colored pencil and straight edge to draw this path on Graph 1. Using the Pythagorean Theorem to determine the distance from point P to point B, calculate the total distance from point A to point P to point B.

   $$\overline{AP} + \overline{PB} = 28.5\ km + \sqrt{(17\ km)^2 + (19\ km)^2}$$
   $$\overline{AP} + \overline{PB} \approx 53.995\ km$$

   c. The crew may choose to travel from point A to point Q to point B. Use a different colored pencil and straight edge to draw this path on Graph 1. Using the Pythagorean Theorem to determine the distance from point A to point Q, calculate the total distance from point A to point Q to point B.

   $$\overline{AQ} + \overline{QB} = \sqrt{(19\ km)^2 + (28.5\ km)^2} + 17\ km$$
   $$\overline{AQ} + \overline{QB} \approx 51.253\ km$$

Directions: Answer questions 1.d.–1.i. in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

d. Suppose point C is located at (0,3). Using a third color, plot the point C(0,3) on segment $\overline{PQ}$. Draw the path of Rover 1 from point A to point C to point B. Discuss with your mission planning team how to calculate the total distance. Write a summary of the plan.

Models will vary. Determine the distances from point P to point C and from point C to point Q, and then use those distances as legs of different right triangles. Calculate the length of the hypotenuse of each of these right triangles and add those lengths together to obtain the total distance.

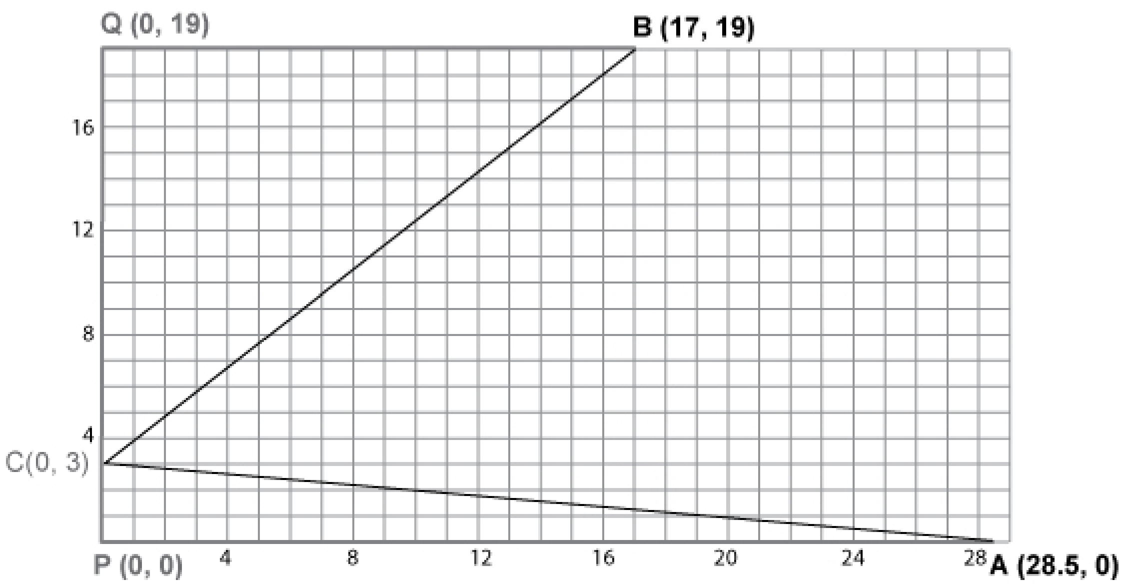


Figure 5: Possible mission graph with the location of point C graphed

e. The Minimal Path Table (Table 1) can be used to assist your team in calculating the total distance from A to any point between point C to point B.
   i. The calculations to determine the total distance for the point discussed in Part 1.d. are shown in the table.
   ii. Discuss the reason for each process column with your team members.
   iii. Working with your mission planning team, select and plot a different location for point C on segment $\overline{PQ}$ that the group thinks will result in a shorter total distance, $\overline{AC} + \overline{CB}$. Complete the process columns in Table 1 for your group's new point.
   iv. Repeat the process with the other three points.

Student table values will vary. Answers can be determined from lists created on the graphing calculator.

Table 1: Minimal Path Table

C (0, n)	$\overline{\mathbf{PC}}$	$\overline{\mathbf{CQ}}$	$\overline{\mathbf{AC}}$	$\overline{\mathbf{CB}}$	$\overline{\mathbf{AC}}+\overline{\mathbf{CB}}$
(0, 3)	*3 km*	*(19 – 3) km*	$\sqrt{(28.5\ km)^2+(3\ km)^2}$	$\sqrt{(17\ km)^2+(16\ km)^2}$	$\sqrt{(28.5\ km)^2+(3\ km)^2}+\sqrt{(17\ km)^2+(16\ km)^2}\approx 52.003\ km$
(0, 7)	*7 km*	*(19 – 7) km*	$\sqrt{(28.5\ km)^2+(7\ km)^2}$	$\sqrt{(17\ km)^2+(12\ km)^2}$	$\sqrt{(28.5\ km)^2+(7\ km)^2}+\sqrt{(17\ km)^2+(12\ km)^2}\approx 50.156\ km$
(0, 11)	*11 km*	*(19 – 11) km*	$\sqrt{(28.5\ km)^2+(11\ km)^2}$	$\sqrt{(17\ km)^2+(8\ km)^2}$	$\sqrt{(28.5\ km)^2+(11\ km)^2}+\sqrt{(17\ km)^2+(8\ km)^2}\approx 49.337\ km$
(0, 15)	*15 km*	*(19 – 15) km*	$\sqrt{(28.5\ km)^2+(15\ km)^2}$	$\sqrt{(17\ km)^2+(4\ km)^2}$	$\sqrt{(28.5\ km)^2+(15\ km)^2}+\sqrt{(17\ km)^2+(4\ km)^2}\approx 49.671\ km$
(0, 19)	*19 km*	*(19 – 19) km*	$\sqrt{(28.5\ km)^2+(19\ km)^2}$	$\sqrt{(17\ km)^2+(0\ km)^2}$	$\sqrt{(28.5\ km)^2+(19\ km)^2}+\sqrt{(17\ km)^2+(0\ km)^2}\approx 51.253\ km$
(0, n)	*n km*	*(19 – n) km*	$\sqrt{(28.50\ km)^2+(n\ km)^2}$	$\sqrt{(17\ km)^2+((19-n)\ km)^2}$	$\sqrt{(28.5\ km)^2+(n\ km)^2}+\sqrt{(17\ km)^2+((19-n)\ km)^2}$

f. Working with your mission planning team, analyze your table. According to your data, where is the best location for point C so that the total distance Rover 1 travels is minimized? What is the distance traveled? Compare your results with those of two other teams. Does there seem to be one best location in order to minimize the path?

Answers based on student's table minimum value and comparison to other teams. If only integer values of n are used, the minimal distance occurs when point C is (0, 12).

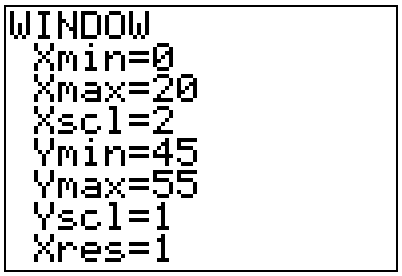
g. Given any point C(0, *n*) along segment $\overline{PQ}$, write an expression in terms of *n* and fill in the last row in Table 1. Use your process from previous rows to draw conclusions about the entries in the last row.

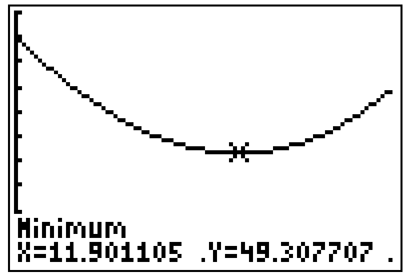
See answer in the last row of Table 1

h. Enter the equation for the total distance into y1 in your graphing calculator. What is the domain for this function? Based on the total distance values in the table, what is an appropriate window for y-values? Graph the equation and use the minimum function of the calculator to approximate the minimum distance.

Domain: $0 \le n \le 19$

The window for the distance values will vary. Student may list any number greater than or equal to 0 or less than or equal to the minimum value for the lower endpoint. Student may list any number greater than or equal to the maximum value, $AP + PB = 28.5\ km + \sqrt{(17\ km)^2 + (19\ km)^2}$ *, for the upper endpoint. The closer the student is to the maximum and minimum values, the better the view window.*





The minimum is when point C is 11.90 kilometers from point Q, which is very close to the answer of 12 (found in Problem 1). The minimum distance of 49.31 kilometers matches the solution found in Problem 1.

i. Show your solution on your mission graph by highlighting Rover 1's path for the shortest distance traveled. How close was your prediction, point G, for the shortest total distance to the actual value found?

Answers vary based on the student minimum from Part 1.f.

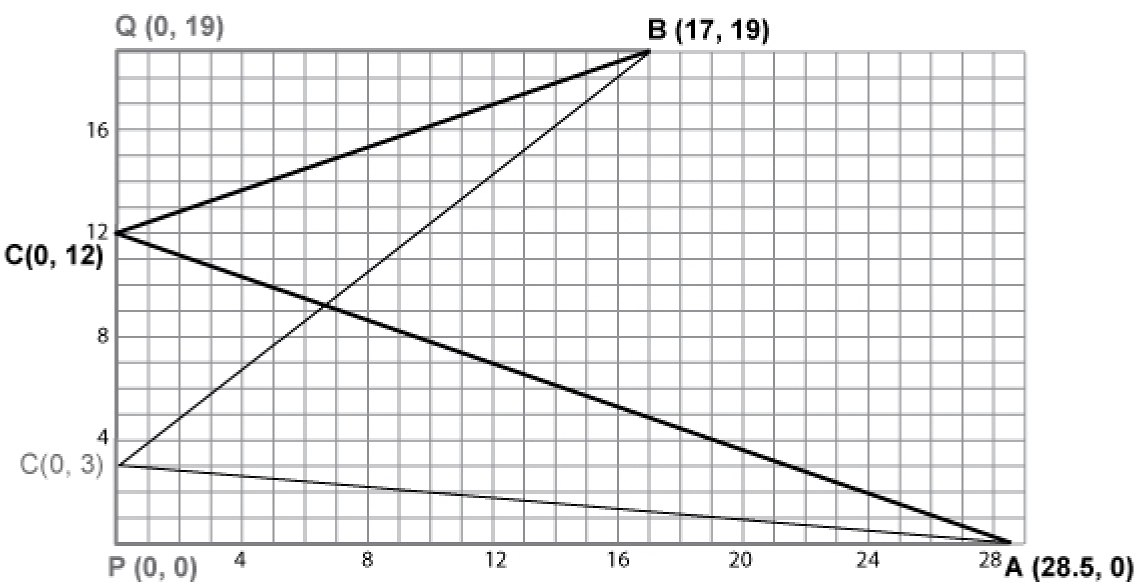


Figure 6: Possible solution for minimal distance shown on a mission graph.

Directions: Answer questions 2.a–2.d in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

2. Minimal Time Problem

   The average speed of Rover 1 is 8.9 kilometers/hour with an empty payload. With a full payload of rocks, the average speed drops to 5.5 kilometers/hour.

   a. Using the locations chosen in Problem 1 and the Minimal Time Table (Table 2), find the time required for each path.

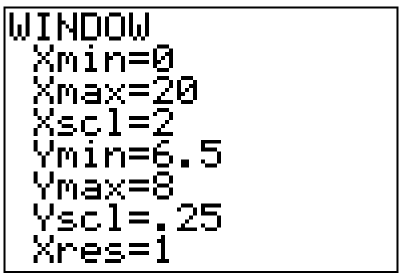
Table 2: Minimal Time Table

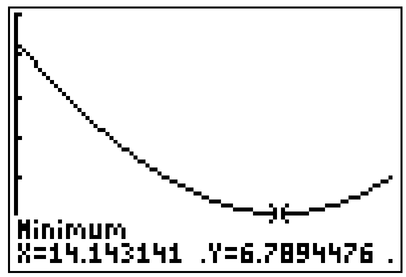
C (0, *n*)	$\overline{\textbf{AC}}$	$\overline{\textbf{CB}}$	**Total Time,** $t = \frac{d}{r}$
(0, 3)	$\sqrt{(28.5\ km)^2 + (3\ km)^2}$	$\sqrt{(17\ km)^2 + (16\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (3\ km)^2}}{8.9 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + (16\ km)^2}}{5.5 \frac{km}{hr}}$ $\approx 7.464\ hr$
(0, 7)	$\sqrt{(28.5\ km)^2 + (7\ km)^2}$	$\sqrt{(17\ km)^2 + (12\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (7\ km)^2}}{8.9 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + (12\ km)^2}}{5.5 \frac{km}{hr}}$ $\approx 7.081\ hr$
(0, 11)	$\sqrt{(28.5\ km)^2 + (11\ km)^2}$	$\sqrt{(17\ km)^2 + (8\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (11\ km)^2}}{8.9 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + (8\ km)^2}}{5.5 \frac{km}{hr}}$ $\approx 6.849\ hr$
(0, 15)	$\sqrt{(28.5\ km)^2 + (15\ km)^2}$	$\sqrt{(17\ km)^2 + (4\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (15\ km)^2}}{8.9 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + (4\ km)^2}}{5.5 \frac{km}{hr}}$ $\approx 6.794\ hr$
(0, 19)	$\sqrt{(28.5\ km)^2 + (19\ km)^2}$	$\sqrt{(17\ km)^2 + (0\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (19\ km)^2}}{8.9 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + (0\ km)^2}}{5.5 \frac{km}{hr}}$ $\approx 6.940\ hr$
(0, *n*)	$\sqrt{(28.50\ km)^2 + (n\ km)^2}$	$\sqrt{(17\ km)^2 + ((19 - n)\ km)^2}$	$\frac{\sqrt{(28.5\ km)^2 + (n\ km)^2}}{8.90 \frac{km}{hr}} + \frac{\sqrt{(17\ km)^2 + ((19-n)\ km)^2}}{5.50 \frac{km}{hr}}$

b. According to your data, where is the best location for point C so that the total time traveled is minimized? What is the minimum time for the mission? Compare your results with those of two other teams. Based on the comparisons, what is the best location for point C to minimize the total time for the mission?

Answers vary based on student's minimum table value and comparisons to other teams. If only integer values of n are used, the minimal time occurs when n is between 14 and 15.

c. Enter the equation for the minimum time into y2 in your graphing calculator. Graph the equation and use the minimum function of the calculator to determine the minimum time. What is the best location for point C so that the total time traveled is minimized? What is the actual minimum time for the mission?





The minimum is when point C is 14.14 kilometers from point Q. The minimum time for this mission is 6.79 hours.

d. Show your solution on your mission graph by highlighting Rover 1's path for the minimum travel time. What is the difference in the minimum time your team determined in Part 1.b. and the actual minimum time?

Answers vary based on student minimum from Part 1.b The solution for minimal time is represented by the dashed line.

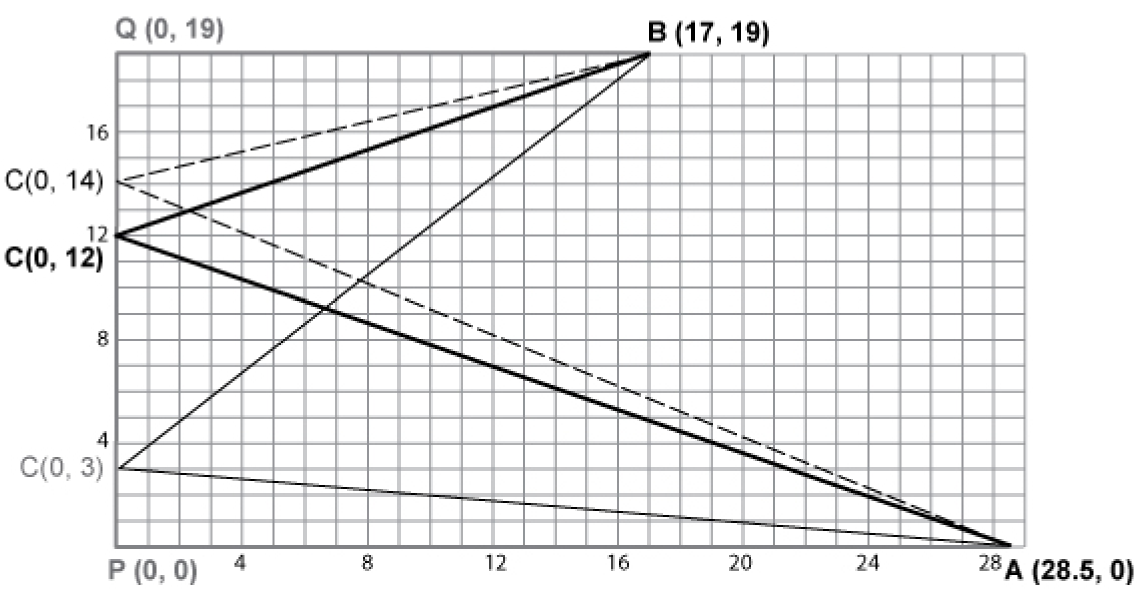


Figure 7: Possible solution for minimal time shown on a mission graph.

Directions: Complete question 3 independently.

3. Now that you have determined both a minimum distance and a minimum time, which would you consider to be the better choice and why?

   Answers will vary. Time would be a better choice if the astronauts' life support was low. Distance would be a better choice if the rover had limited fuel.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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